

# **Investigation of Model Sensitivities and Model Errors with Relation to Data Assimilation Systems**

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## **LONG-TERM GOALS**

To investigate basic predictability issues related to model sensitivities to uncertainties in the initial and boundary conditions; how these sensitivities are dependent on different model dynamics, different data types and data distribution; how their assessment can lead to the evaluation of model errors as intrinsic component of the data assimilation method used.

## **OBJECTIVES**

- a) To assess the dynamical mechanisms that ultimately limit the predictability of a flow system.
- b) To assess uncertainty in models by evaluating estimates of model error covariances from the assimilation method used.

## **APPROACH**

1) The Tropical Atlantic Circulation estimated from Altimetry Data with a Reduced-Rank Stationary Kalman Filter.

Dr. Mark Buehner was a Postdoctoral Associate under the present grant during the period July 1, 2000 – June 30, 2001. The project was completed after he left and the related paper was submitted in April 2002.

A reduced-rank stationary Kalman filter is applied to a realistic model of the tropical Atlantic ocean. The goal is to estimate the sub-surface circulation and thermal structure for studies of the circulation pathways in the Atlantic subtropical and tropical gyres by assimilating TOPEX/POSEIDON sea surface height (SSH) data.

The model is a reduced gravity, primitive equation GCM of the upper ocean with a variable-depth oceanic mixed layer and a domain covering the Atlantic ocean between 30°N and 30°S. Wind stress and heat flux, calculated from wind speed and cloud cover provided by NCEP, are used to force the model at the surface. The assimilation scheme is an approximation to the extended Kalman filter in which the error covariances of the state estimates are only calculated in a reduced-dimension subspace spanned by a small number of EOFs. Results from previous studies concerned with assimilating SSH in the tropical ocean suggest that the cost process of dynamically evolving the error covariances only result in minor improvement to the state estimates. Therefore, to obtain an assimilation procedure,

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which only requires slightly more computational efforts than simple model integration, the asymptotically stationary error covariances are used.

Assimilation of simulated SSH data in twin experiments demonstrates the ability of the method to successfully constrain the circulation and subsurface thermal structure. Assimilation of actual TOPEX/POSEIDON altimetry data resulted in a 23.6% reduction in the rms misfit with observed SSH relative to pure model integration. Also, the agreement between the power spectra of the observed and model SSH is significantly improved by the assimilation. Evaluation of the impact on the subsurface fields is more difficult due to a lack of independent measurements. However, changes in the thermocline structure appear reasonable and the correlation between the observed SSH and the depth of the model thermocline are improved by the assimilation.

2) A comparison of assimilation results from the ensemble Kalman filter and the reduced rank extended Kalman filter.

Dr. Xiaoyun Zhang joined MIT on April 1, 2001 and has been supported by the present grant since then as Postdoctoral Associate.

The goal of this study is to evaluate the performances of the ensemble Kalman filter and the reduced-rank extended Kalman filter when applied to different dynamic regimes. Data assimilation experiments are performed using an eddy-resolving quasi-geostrophic model of the wind-driven ocean circulation. By changing the eddy viscosity, this model exhibits two qualitatively distinct behaviors: strongly chaotic for the low viscosity case and quasi-periodic for the high viscosity case.

In the reduced-rank extended Kalman filter algorithm, the model is linearized with respect to the time-mean from a long model run without assimilation; the reduced state space is obtained from a small number of leading empirical orthogonal functions (EOFs) derived from the same long model run; and it is assumed that a steady state limit exists of the error covariances. The ensemble Kalman filter is based on estimating the forecast error statistics using Monte Carlo methods.

The results show that for both strongly chaotic and quasi-linear cases, about 32 ensemble members are sufficient to accurately describe the non-stationary, inhomogeneous and anisotropic structure of the forecast error covariance. For the strongly chaotic case, the performance of the reduced-rank extended Kalman filter is very similar to the simple optimal interpolation and the ensemble Kalman filter greatly outperforms the reduced-rank extended Kalman filter (figure 1). For the quasi-periodic case, both the reduced-rank extended Kalman filter and the ensemble Kalman filter are able to significantly reduce the analysis error and their performances are similar (figure 2).

3) Dr. Xiaoyun Zang has begun to construct the ensemble Kalman filter for the primitive equation OGCM of point 1, for which the reduced rank K-filter is already available. The goal is to compare the two schemes in a fully realistic simulation of the Atlantic Ocean circulation assimilating the TOPEX altimetric data.

## **WORK COMPLETED**

- 1) Construction of the reduced rank K-filter for the primitive equation GCM.
- 2) Construction of the ensemble K-filter for the double-gyre wind-driven model.

## RESULTS

Discussed under “APPROACH”.

## PRESENTATIONS

M. Buehner and P. Malanotte-Rizzoli, “The tropical Atlantic ocean circulation estimated from altimetry data with a reduced-rank stationary Kalman filter”, American Meteorological Society, Orlando, Fla., January 2002.

P. Malanotte-Rizzoli, “Recent experience with practical data assimilation”, American Meteorological Society, Orlando, Fla., January 2002, invited paper.

P. Malanotte- Rizzoli and X. Zang, “A comparison of assimilation results from the Ensemble Kalman filter and the reduced-rank Kalman filter”, European Geophysical Society XXVII General Assembly, Nice, France, April 2002, invited paper.

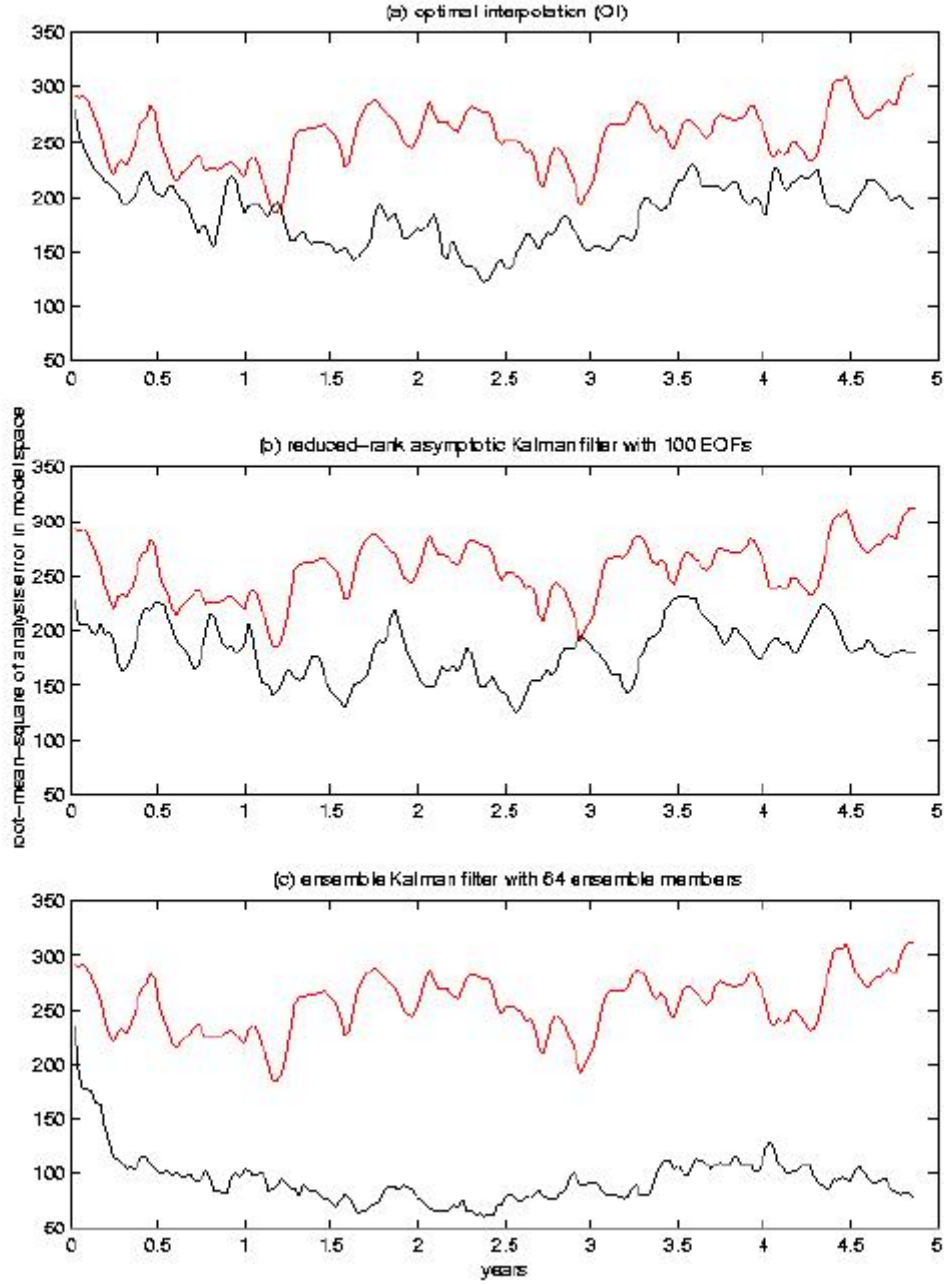
## PUBLICATIONS

Mahadevan, A, J. Lu, S.P. Meacham, and P. Malanotte-Rizzoli, 2001, “The predictability of large scale wind-driven flows”, *Nonlinear Processes in Geophysics*, 8, 449- 465.

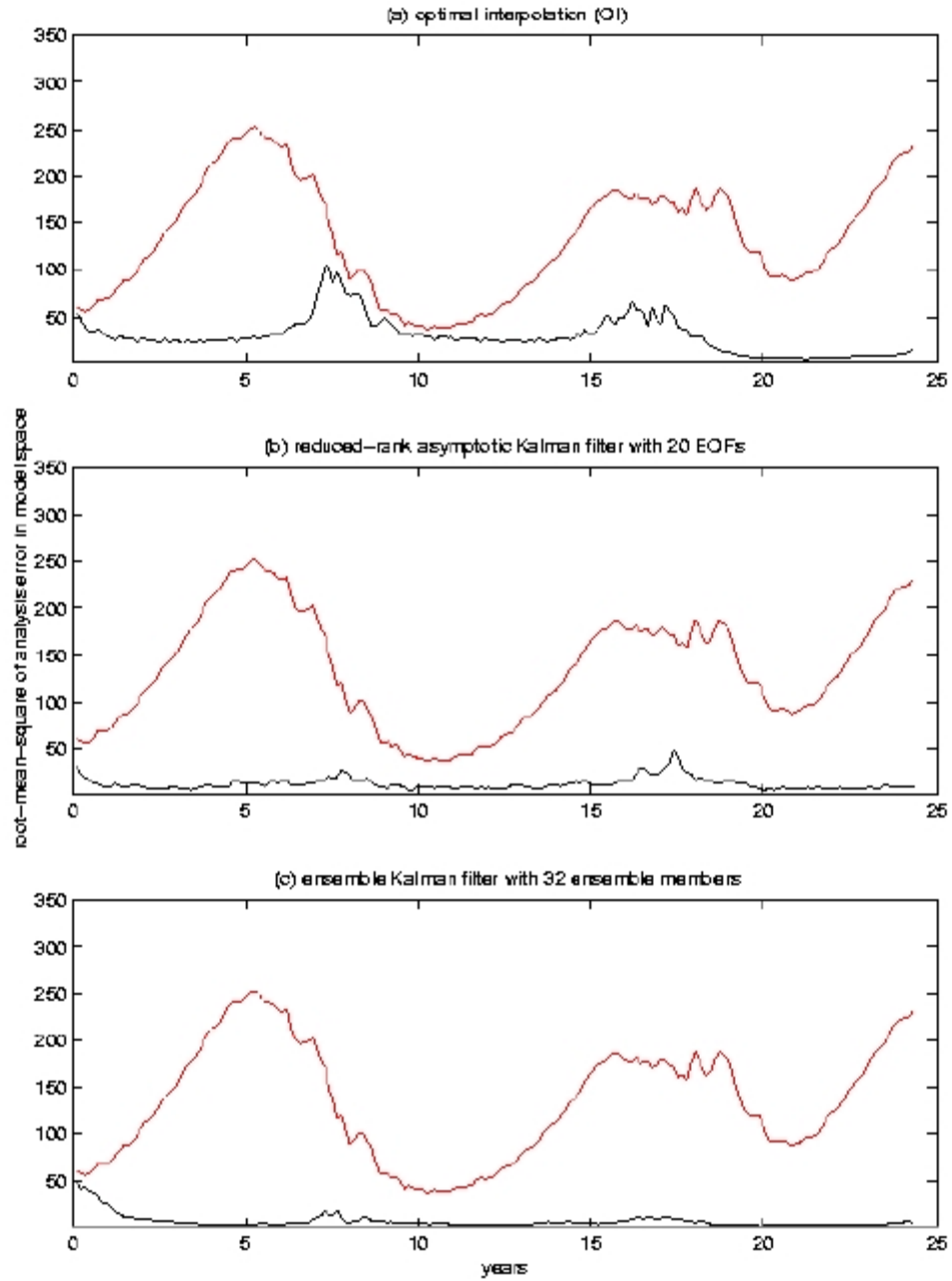
M. Buehner and P. Malanotte-Rizzoli, 2001, “Reduced-rank Kalman filters applied to an idealized model of the wind-driven ocean circulation” in press *J. Geophys. Res.*, 2002.

M. Buehner, P. Malanotte-Rizzoli, T. Inui, and A.J. Busalacchi, 2002, “The tropical Atlantic ocean circulation estimated from altimetry data with a reduced-rank stationary Kalman filter”, under review *J. Geophys. Res.*

X. Zang and P. Malanotte-Rizzoli, “A comparison of assimilation results from the ensemble Kalman filter and the reduced-rank extended Kalman filter”, submitted to *Nonlinear Processes in Geophysics*, 2002.



**Figure 1. Unconstrained model forecast error (red line) and analysis error (black line) every 9 days during 5-year assimilation cycle for the strongly chaotic case from the assimilation experiment of (a) optimal interpolation, (b) reduced-rank asymptotic Kalman filter with 100 EOFs and the tuning parameter  $\gamma=0.0025$ , and (c) ensemble Kalman filter with 64 ensemble members and the “influence” radius  $r_0=128$ .**



**Figure 2.** Unconstrained model forecast error (red line) and analysis error (black line) every 45 days during 25-year assimilation cycle for the quasi-periodic case from the assimilation experiment of (a) optimal interpolation, (b) reduced-rank asymptotic Kalman filter with 20 EOFs and the tuning parameter  $\gamma=0.04$ , and (c) ensemble Kalman filter with 32 ensemble members and the “influence” radius  $r_0=128$ .